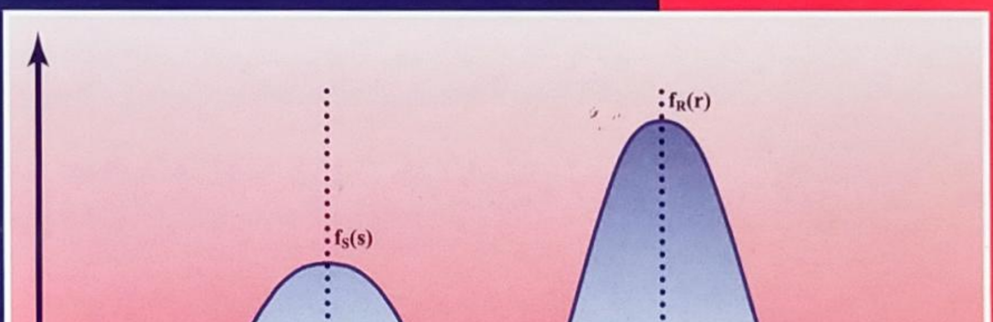
Probability, Reliability and Statistical Methods in Engineering Design



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Probability, Reliability, and Statistical Methods in Engineering Design

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Dedication

To my wife Carolyn for her help in all phases of the development of this book, our son Justin, and my mother Angur Haldar.

Achintya Haldar

To my parents, Ganesan Sankaran and Janaki Sankaran, and my wife Monica.

Sankaran Mahadevan

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Preface

The application of risk and reliability in the analysis, design, and planning of engineering systems has received worldwide acceptance from the engineering profession. As a result of extensive efforts by different engineering disciplines during the last three decades, design guidelines and codes are being modified or have already been modified to incorporate the concept of risk-based analysis and design. The Accreditation Board of Engineering and Technology (ABET) now requires that all civil engineering undergraduate students in the U.S. demonstrate knowledge of the application of probability and statistics to engineering problems, indicating its importance in civil engineering education.

There are many motivations behind this book, and they need some elaboration here. In the 1970s, while deeply involved with safety issues for nuclear power plants, the first author observed that ordinary engineers were not trained in risk or reliability-based analysis and design procedures. This is true even today to some extent. Many engineers are reluctant to use these procedures because reliability methodology appears to be difficult and mathematically demanding. The available literature is not easy to read, and the basic concept is buried in complex mathematical notations, symbols, and diagrams. Information is available on a piecemeal basis emphasizing a few areas of interest to the writers, and the numerous articles fail to give a broad, comprehensive understanding of the current state of the art. Some of the research issues and items of interest to academicians are emphasized; however, some basic concepts that might facilitate practical understanding and implementation are overlooked or deemphasized.

For engineers who understand risk-based analysis and design, applying the concept to engineering problems still appears to be very difficult. This is because the basic theories and algorithms have been under considerable research, development, and verification efforts until recently. There has been much ferment in this field during the past two decades, resulting in numerous publications with various methods and theoretical arguments. However, much experience has been gained from this research during the past decades, and now it is time to crystallize these concepts into a simple, ready-to-use format to enable risk-based design. This is an important motivation behind this book.

In this book we strive to deal comprehensively with issues relevant to students, professionals, and society. There are several target audiences for this book. The book addresses issues of interest to undergraduate and graduate students of civil engineering, and to other engineering disciplines to a lesser extent; it will also interest practicing engineers with little or no background in probability and statistics, and the general risk and reliability research community. The material is presented attractively and efficiently and is based on a great deal of experience gained by dealing with students and practicing engineers on safety-related issues.

To satisfy the objectives of the book and the target audiences, we develop the material gradually, first addressing very simple concepts and later moving to advanced concepts required to implement risk-based design concepts in practical problems. The statistical knowledge required to implement the concept is presented in an optimal way, fitting it into the overall scheme of risk or reliability evaluation of engineering systems. Many reliability analysis methods with various degrees of complexity and sophistication are presented. The area of simulation is becoming an integral part of risk and reliability analysis, even with an elementary knowledge of probability and statistics. The extensive use of personal computers is the motivation behind the chapter on simulation techniques. Simulation also plays an important role in verifying the results obtained using more mathematically demanding analytical methods.

The material is developed primarily for undergraduate engineering students; however, it can be used by graduate engineering students as well and by practicing engineers with no formal instruction on the subject. The material is presented directly and straightforwardly with many practical examples, in an attempt to increase appreciation for the subject and demystify the complicated mathematical theory of risk and reliability. An understanding of the contents of the book is expected to provide a sufficient working knowledge of risk and reliability to all interested parties. It is expected that these chapters can be taught in one semester and will satisfy the ABET requirements.

The book can be used to teach any required undergraduate course for engineers on probability and statistics. These chapters are developed so as to be understandable by members of any engineering discipline. Chapter 1 presents the basic concepts of risk and reliability. Chapters 2 through 8 start with the fundamentals of probability and statistics and present the concepts of reliability analysis, with applications to simple components and systems. The mathematical aspects of evaluating risk and reliability (i.e., set theory) are discussed in Chapter 2. Chapter 3 discusses methods to quantify randomness in terms of data collection and analysis for one or multiple random variables and correlations among random variables. Risk evaluation procedures using some of the common distributions including extreme value distributions are presented in Chapter 4. Statistical concepts regarding the selection of suitable distributions and the estimation of parameters to uniquely describe a selected distribution are discussed in Chapter 5. Chapter 6 discusses several methods for evaluating uncertainty in the response of engineering systems using information on the random variables involved in the problem. Regression analysis procedures are also presented in this chapter. The concept of reliability analysis and currently available risk evaluation procedures are discussed in Chapter 7. Advanced reliability analysis topics are presented in Chapter 8. The use of simulation in estimating risk is introduced in Chapter 9. These chapters are self-contained and can be used in an undergraduate course to satisfy the ABET requirements. Additional materials required to make the book self-sufficient are provided in six appendixes.

This book discusses all the fundamentals of reliability and statistics required for risk-based engineering analysis and design. It primarily emphasizes reliability analysis procedures when the functional relationship among the load and resistance-related variables (i.e., the performance function or the limit state equations) is available in explicit form. The book briefly addresses reliability evaluation procedures when the performance functions are implicit; we discuss this more completely in another book, *Reliability Assessment Using Stochastic Finite Element Analysis*, published by John Wiley & Sons in 2000.

To improve the readability of the book, citations in the middle of a discussion are avoided. Many people contributed to the development of risk and reliability-based engineering analysis and designs, and an extensive list of references is given at the end of the book to acknowledge their contributions. We have tried our best to make this list as complete as possible. We also would like to thank the many anonymous reviewers for their constructive comments and suggestions. Their help was essential in developing this book.

Mr. Jungwon Huh's help in developing figures, tables, and numerical solutions for many problems given in the book is very much appreciated.

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NOTES FOR INSTRUCTORS

The book is suitable for a sophomore or junior level course as well as for a senior or first-year graduate level course. Based on extensive teaching experience, we would like to suggest a tentative course outline for the two courses.

Sophomore or Junior Level Course (Average Duration 15 Weeks)

This outline is applicable when the students have very limited exposure to set theory. For this class, the material in Chapters 1 to 3 needs to be covered in detail and can be covered in 10 lectures. Some of the commonly used continuous and discrete random variables discussed in Chapter 4 can then be introduced. This topic can be covered in 9 lectures. Section 4.5 on extreme value distributions can be skipped if desired. The discussion in Chapter 5 on determining the underlying distribution for a given set of data or observations, and the point and interval estimations of its parameters, can be covered in 6 lectures. Estimating randomness in response variables including the regression analysis can be covered in 6 lectures. The concept of reliability evaluation, presented in Chapter 7, needs to be covered in detail and may take over 8 lectures. The advanced topics of reliability analysis, discussed in Chapter 8, can be skipped for an introductory undergraduate course. The chapter on simulation, Chapter 9, may take about 4 lectures. These recommendations cover about 43 lectures. Two additional lecture hours are left for examinations or other activities.

Senior or First-Year Graduate Level Course (Average Duration 15 Weeks)

It is expected that these students are familiar with set theory and have some basic knowledge on statistics, and these topics only need to be reviewed. We suggest covering the first 3 chapters in 6 lectures. Chapter 4, with more discussion on Section 4.5, can be covered in 8 lectures. Chapter 5 needs about 6 lectures. About 6 lectures can be spent on Chapter 6. Chapters 7 and 8 need to be covered in detail and may take over 12 lectures. Chapter 9 can be covered in 5 lectures. Again, this lecture guide covers 43 lecture hours; an additional 2 lecture hours are left for examinations and other activities.